Tutorial 4 – Giovanni Filomeno - 12315325

# Exercise 18

## Homogeneous Poisson Process

My code for generating a path of the homogeneous Poisson process is presented in Figure 1.

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| Figure : Homogeneous Poisson process |

## Plot a path for and

Figure 2 represents my result for the Poisson path.

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| Figure : Homogeneous Poisson path for and |

## Empirical pmf vs simulated of random variable

My algorithm for comparing the two pmf is presented in Figure 3, while the obtained result is in Figure 4. The empirical probability mass function (pmf) of , obtained via Monte Carlo simulation, closely matches the theoretical Poisson pmf with parameter . This agreement confirms the validity of the simulation approach in approximating the distribution of the Poisson process at time .

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| Figure : Empirical pmf vs simulated algorithm – ex18c |

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| Figure : Empirical pmf vs simulated results – ex18c |

## Empirical pmf vs simulated of jumping time

My algorithm for comparing the two pmf is presented in Figure 5, while the obtained result is in Figure 6. Also in this case, the empirical distribution of the fourth jump time aligns closely with the theoretical Gamma distribution, confirming the known result that the -th jump time in a Poisson process follows a Gamma distribution with shape and rate .

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| Figure : Empirical pmf vs simulated algorithm – ex18d |

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| Figure : Empirical pmf vs simulated results – ex18d |

# Exercise 19

1. Algorithm for non-homogeneous Poisson process

My code for generating a path of the non-homogeneous Poisson process is presented in Figure 7.

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| Figure : Non-homogeneous Poisson process |

1. Plot a path for

The requested path is presented in Figure 8.

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| Figure : Non-homogeneous Poisson path for |

1. Empirical vs Simulated pmf

In this case, the empirical vs simulated pmf presents a shift. This is an expected results when oscillates. Figure X shows the comparison.

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| Figure : Empirical pmf vs simulated results – ex19c |

# Exercise 20

1. Path of Wiener process

My code for simulate the Wiener process is presented in Figure 10.

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| Figure : Wiener process algorithm |

1. 5 Wiener process

Figure 11 shows five independent sample paths for , over with , and the red straight line is the theoretical mean .

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| Figure : Five paths Wiener process |

1. 5 Wiener process with different drift

Figure 12 presents one long path for each drift value (all with ) over . All curves share the same Brownian variability but diverge according to their deterministic drift.

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| Figure : Five paths Wiener process with different drifts |

1. 5 Wiener with different

The last figure (Figure 13) shows the requested simulation varying sigma.

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| Figure : Five paths Wiener process with different sigma |

# Exercise 21

1. Geometric Brownian motion

My code for simulating the path of the geometric Brownian motion is presented in Figure 14.

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| Figure : Geometric Brownian motion |

1. 10 different Brownian paths

Figure 15 shows 10 independent geometric Brownian motion paths starting at .

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| Figure : Different GBM paths |

# Exercise 22

1. Euler-Maruyama method

Figure 16 presents the code and the resulting plot for the Euler-Maruyama method.

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| Figure : Euler-Maruyama code and plot | |

1. Milstein method

Figure 17 presents the code and the resulting plot for the Milstein method.

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| Figure : Milstein code and plot | |

# Exercise 23

1. Reproducing the Week10 graph

Figure 18 represents a mimicking graph of Week10. A small shift in y is given to perfectly match the reference graph.

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| Figure : Week10 Graph mimic |

# Exercise 24

I decided to combine the entire exercise in one. Using the numerical integration, I obtained while using the Monte Carlo with I obtained . Figure 19 shows how the estimation of changes along the realization . The red line represents the value of obtained via standard integration.

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| Figure : Standard integration vs Monte Carlo |